A general S-bend approximation by cascading multiple sections of uniformly curved waveguides paper

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Journal of Physics: Conference Series

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To cite this article: Ary Syahriar *et al* 2019 *J. Phys.: Conf. Ser.* **1196** 012030

A General S-bend Approximation by Cascading Multiple Sections of Uniformly Curved Waveguides

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Abstract. Curved waveguides are an important wave guiding structure and have been widely used in building integrated photonics circuits with various functions including directional couplers, modulators, ring resonators, and Mach-Zehnder interferometers etc. However, curved structure always leads to attenuation of the guided mode as it propagates through the bend region. In practice the bending loss may be negligible if the radius of curvature is large, but the loss increases rapidly at small radii. Amongst curved waveguide structures the S-bend has been widely used, because it is relatively easy to design and can provide a low transition loss between parallel offset waveguides. In this paper we present a new general approach to S-bend optical waveguides by cascading multiple sections of uniformly curved waveguides. The aims are to offer maximize structure that can be used in more powerful beam propagation methods to estimate the loss in bends of continuously-varying curvature. We approach the S shaped bend waveguide by multiple sections such as 2, 6, 14 and 30 sections back to back and calculate the total loss. The convergence of resulting curvature variation are very rapid and the overall loss calculation found good agreement with analytical calculation based on low slope approximation.

1. Introduction

Waveguide bends are required in many basic optical structures, including directional couplers, modulators, ring resonators, and Mach-Zehnder interferometers [1]-[4]. Further application in photonic circuits has also been demonstrated such as single wavelength ring lasers [5]–[6], modulators, add-drop filters [7]-[9]. The main problem of curved waveguide is the power loss due to radiation as it passes through curve section. Radiation loss can be reduced by decreasing the curvature of the bend or by increasing the confinement of the modal field. However, these changes generally result in either an increase in the overall device length or an increase in the coupling loss when optical fibers are connected at the input and output of the circuit. Therefore, a number of different alternatives have been proposed for reducing radiation loss, such as the use of an offset waveguide junction [7], an S-shaped bend [8]-[10] and an optimized continuous path [11]. Correct design of these structures requires accurate characterization of field shapes and shape changes along the bend. Amongst them the S-bend has been widely used, because it is relatively easy to design and can provide a low transition loss between parallel offset waveguides. We concentrate on S-shaped waveguide bend geometries and their analytic approximations. To construct a smooth S-shaped transition connecting two parallel offset guides, several methods have been used. For example, Marcuse used an electron beam machine to cascade many sections of a straight guide in staircase geometry to approximate a continuous bend [11]. The staircase section was assumed to be a guide with periodically distorted axis. Losses obtained using this technique was found to depend on the period of the periodic structure; typically, the loss decreased as the period increased. Another method, described by Taylor, used coherent coupling between closely-spaced abrupt bends [14]. The entire bend comprised a number of equal-length straight waveguide segments, each separated by an abrupt change in angle from its predecessor [13]. Using this technique, light coupled from a guided mode into an unguided mode at each discontinuity could be coupled back into the guided mode, provided appropriate phase relationships were maintained between the radiations coupled from adjacent sections [13]. In this paper we proposed a new approximation to the S bend by joining together N segments of curve waveguides with constant radii of curvature. The general aims are to find simple methods of predicting the loss in bends whose functional form is described by a 'sinusoidal shape function' y = f(x), where f is continuous in its first derivative. This method of approximating a sinusoidal bend can clearly be applied to other arbitrary bend geometries, merely provided they are continuous in their first derivative. Its advantage is that it allows the use of more powerful beam propagation methods based on polar co-ordinates to estimate the loss in bends of continuously-varying curvature. Accurate approximation to the true geometric shape using such a small number of sections suggests that rapid convergence will be obtained in numerical integration of the loss using this approach [16].

2. Research Method

Figure 1 shows a schematic diagram of a sinusoidal bend of length *L* connecting two parallel guides that are offset by a transverse distance *l*.

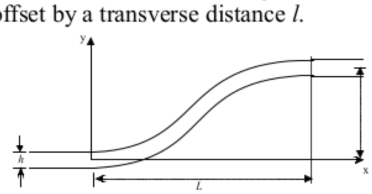

Figure 1. Schematic diagram of an S-shaped waveguide bend with a transition length L and a lateral offset l. [16]

The lateral position *y(x)* of the waveguide at a distance *x* along the bend is described by the following expression [9][15]:

$$y(x)=\frac{xl}{L}-\frac{l}{2\pi}\sin\left(\frac{2\pi x}{L}\right) \tag{1}$$

This particular form of transition has been found to be effective in achieving low loss, because it eliminates discontinuities in *y*, *y'* (the first derivative) and *y''* (the second derivative) of the function. To find the radius of curvature of such a function, two different expressions are often used. The first expression is the mathematically exact radius of curvature, which is defined as [9]:

$$r(x)=\frac{(1+y'(x)^2)^{\frac{3}{2}}}{y''(x)} \tag{2}$$

Due to the presence of the radical, Equation (2) is unfortunately too complicated for general use. As a result, an alternative 'low slope approximation' is often used instead; this can be obtained by assuming that $1+y(x)^2\approx 1$ in Equation (2), so that:

$$r(x)\approx\frac{1}{y''(x)} \tag{3}$$

For the sinusoidal shape function of Equation (1), the low-slope approximation is valid when $L >> l$ (which is typically the case for a bend in a weakly-confining waveguide). In this case, the curvature $1/r$ is given by:

$$\frac{1}{r}\approx\frac{2\pi l}{L^2}\sin\left(\frac{2\pi x}{L}\right) \tag{4}$$

Equation (4) implies that the curvature of a sinusoidal bend is variable, with zero curvature at the input and output and a maximum curvature (and hence a minimum bend radius) at $x=L/4$ and $x=3L/4$.

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To simplify the geometry, other approximations are often used for S-shaped bends particularly that based on N circular arc sections of constant radius of curvature. Figure 2 shows the general geometry of approximation of waveguide bend by cascading multiple sections of uniformly curved guide. Here, the bend is approximated by joining together N segments of constant radii of curvature. The i^{th} section has radius r_i, is centered at the point (x_{ci}, y_{ci}), and subtends an angle from φ_{i-1} to φ_i. The unknown quantities r_i, x_{ci}, y_{ci} and φ_i may be obtained from the requirement that the approximate structure must intersect the true bend geometry at each point (x_i, y_i), and be continuous in its first derivative.

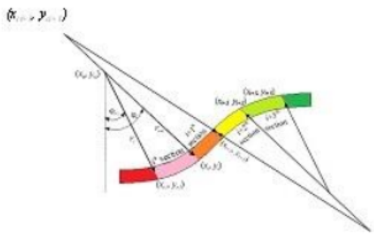

Figure 2. Approximation to a general waveguide bends by cascading multiple sections of uniformly curved guide.

From the first condition, we obtain:

$$x_{i-1} = x_{ci} + r_i sin(\varphi_{i-1}) \quad (5)$$

$$x_i = x_{ci} + r_i sin(\varphi_i) \quad (6)$$

$$y_{i-1} = y_{ci} - r_i cos(\varphi_{i-1}) \quad (7)$$

$$y_i = y_{ci} - r_i cos(\varphi_i) \quad (8)$$

Subtracting Equation (5) from Equation (6), and subtracting Equation (7) from Equation (8) we obtain:

$$(x_i - x_{i-1}) = r_i\{sin(\varphi_i) - sin(\varphi_{i-1})\} \quad (9)$$

$$(y_i - y_{i-1}) = r_i\{cos(\varphi_{i-1}) - cos(\varphi_i)\} \quad (10)$$

Equations (9) and (10) can then be combined to obtain:

$$\frac{x_i - x_{i-1}}{y_i - y_{i-1}} = \frac{sin(\varphi_i) - sin(\varphi_{i-1})}{cos(\varphi_{i-1}) - cos(\varphi_i)} \quad (11)$$

Substituting the values (x_1, y_1) and (x_0, y_0) and $\varphi_0 = 0$, obtained from the true sinusoidal curve, Equation (11) may be solved numerically to obtain φ_1. A suitable algorithm is the method of bisection, which may obtain an accuracy of one part in 106 after 20 iterations. Values of x_{c1}, y_{c1} and r_1 may then be obtained from Equations (5)-(6). A similar procedure can then be applied to find φ_2, (x_{c2}, y_{c2}) and r_2, and so on, until the end of the bend is reached. Clearly, there is some freedom in the choice of the number of sections, and in their start and end positions. In the case of a sinusoidal bend, the curvature variation predicted by the low slope approximation in Equation (4) contains symmetries at $x=\frac{L}{4}$ and $x=\frac{3L}{4}$. For the circular arc approximation, assuming segments of equal length measured in the x-direction, similar symmetries are obtained when N is twice an odd number, so that $N=2\{2j+1\}$, where j is an integer. For example, when $j = 0$, $N = 2$ (leading to the two section approximation discussed earlier); when $j = 1$, $N = 6$; when $j = 2$, $N = 10$, and so on. Higher numbers in the series include $N = 14$, 18, 22, 26 and 30.

2.1. *Estimating Loss In Waveguide Bends*

We now consider how the attenuation coefficient α is used to calculate the loss occurring in a bend of uniform curvature. Marcatili and Miller have shown that the attenuation coefficient is indeed constant for a fixed radius, and can be expressed as [1]:

$$\alpha = C_1 e^{-C_2 r} \quad (12)$$

where C_1 and C_2 are functions of the waveguide parameters but are independent of r. In this case, the modal power at distance s is $P_s = P_0 exp(-\alpha s)$ where P_o is the input power. The total loss in dB is then [16]:

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$$loss = \frac{10}{log_{e(10)}}(-\alpha s)(dB) \tag{13}$$

Using Equation (13), the radiation loss of a single section of uniformly curved step-index planar waveguide can be computed. By extending this simple theory, we can easily calculate the losses obtained when *N* bends of different radii are cascaded. In this case, the bending loss can be written as a summation of the form:

$$loss = \frac{10}{log_{e(10)}}\left(\sum_{i=1}^{N} \alpha_i s_i\right)(dB) \tag{14}$$

Where α_i and s_i are the absorption coefficient and length of the i^{th} section, respectively. As the number of cascaded sections tends to infinity, and their length tends to zero, the summation of Equation (14) gives way to a line integral of the form [9]:

$$loss = \frac{10}{log_{e(10)}}\int_0^S \alpha(s)ds\,(dB) \tag{15}$$

Here's s represent the local position along the bend, $\alpha(s)$ is the attenuation coefficient at that point, and S is the total length of the bend. Although Equation (15) appears deceptively simple, it is surprisingly difficult to evaluate the line integral analytically. By substituting Equation (2) into Equation (14), and using low slope approximation, the loss is given by [16]:

$$loss = \left\{\frac{10}{log_{e(10)}}\right\}\int_0^L C_1 exp(-C_2 r(x))\,dx(dB) \tag{16}$$

No analytic solution to this equation is known, although it is amenable to numerical integration using e.g. Simpson's method.

3. Results and Discussion

To get a better understanding of the different approximations to the shape of a sinusoidal S-bend, Figure 3 compares the variation of curvature with distance predicted by i) the continuous analytic approximation, ii) a discrete approximation based on two back-to-back circular arc sections, and iii) a similar discrete approximation based on six circular arc sections. All values are normalized with respect to the highest curvature obtained in the continuous analytic approximation. For the continuous analytic approximation, the curvature variation is the expected sinusoidal function. The two-section structure represents the most elementary discrete approximation to a sinusoidal bend, and has a square-wave curvature variation.

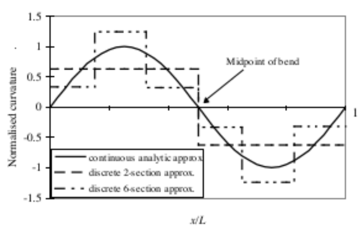

Figure 3. Curvature of a sinusoidal bend as a function of normalized distance for i) the constant analytic approximation, ii) a discrete 2 section approximation, iii) a discrete 6 section approximation.

Clearly, improved agreement can be achieved by using six sections rather than two, when a staircase approximation to the curvature variation is obtained. Even better approximations are possible by cascading a greater number of sections, as shown in Figure 4. It can be seen that the curvature variation of a fourteen-section approximation closely resembles that of the true sinusoidal bend, but the discontinuity occurring at the junction between a small number of the sections is still significant. However, by the time the number of sections has risen to 30, the discontinuities are extremely small. This method of approximating a sinusoidal bend can clearly be applied to other arbitrary bend geometries, merely provided they are continuous in their first derivative. Its advantage is that it allows the use of more powerful beam propagation methods based on polar co-ordinates to estimate the loss in bends of continuously-varying curvature. Accurate approximation to the true geometric shape using such

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a small number of sections suggests that rapid convergence will be obtained in numerical integration of the loss using this approach.

The total loss of such geometry can then be calculated as a summation rather than an integral, using Equation (15). Here, the parameters of n_1=1.463, n_2=1458 =1.525 μm, with h=5 μm, have been used. Figure 5(a) illustrates the loss obtained in this way using, i) 2, ii) 6, iii) 10, iv) 14 sections. Additionally, Figure 5(b) illustrates the loss obtained using v) 18 and vi) 22 sections. As a reference, we compare the results with a numerical integration of the exact equation. For the two-section approximation, the geometry shown in Figure 2 has been adopted. The calculation is performed by substituting a radius of curvature and an angle φ obtained from Equation (11) into Equation (14). The radiation loss obtained is

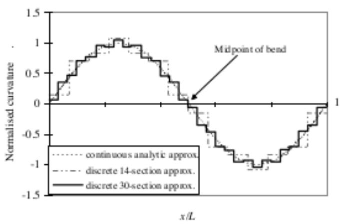
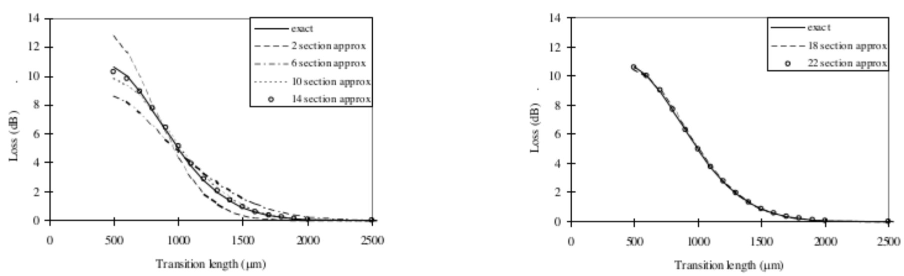


Figure 4. Approximations to an S-shaped sinusoidal bend obtained using multiple sections of curved guide: i) 14 sections, and ii) 30 sections.

clearly higher than that of the exact equation, especially at lower transition lengths, although both results are similar at a longer transition length.



(a) (b)

Figure 5. Bending loss as a function of transition length for multiple-section approximations to sinusoidal S-bends (a) 2, 6 and 14 section approximations. (b) 18 and 22 section approximations

For a six-section approximation, a better improvement is clearly achieved, especially at lower transition lengths, due to the closer match to the real bend geometry. Further improvement can be achieved by using 10 or 14 sections as shown in Figure 5 (a). However, at very short transition lengths, both approximations are still not in very good agreement with the exact equation. However, excellent agreement can be obtained by cascading only a few more sections, for example 18 or 22 sections as shown in Figure 5 (b). To illustrate the accuracy of the multiple-section approximation, Figure 6 shows the convergence of the fractional error in predicted loss as a function of the number of sections, at a fixed transition length of 500 μm. It shows that as a number of sections increases, the accuracy improves very rapidly. For example, for 2 sections the error is 16%, however the error is reversed to -23% as a number of sections is increased to 6; by the time the number has risen to 22, the error is negligible. These results reinforce the earlier estimate of the rapid convergence likely to be obtained by the multiple-section approximation.

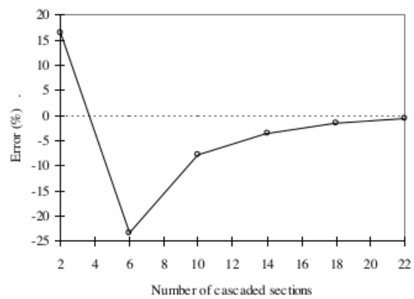


Figure 6. The numerical convergence of multi section approximations.

4. Conclusions

In conclusion, we have examined a different approach to the calculation of the radiation loss in continuously-varying S-shaped waveguide bends. We have shown that the structures themselves can be approximated by cascading a small number of bends with constant curvature, and have presented a suitable approach for deriving the geometric parameters of each section. The convergence of the resulting curvature variation is extremely rapid, suggesting that the convergence of beam propagation solutions based on cascaded sections in polar co-ordinate will also be very fast. Furthermore, the overall loss calculation of cascaded bends with constant curvature found good agreement with calculation results using the method of the low slope approximation with the local loss coefficient, and have verified the expected rapid convergence in modelling continuously-varying S-bends using cascaded section approach.

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