# Analysis of three paralel waveguides using coupled mode theory and the method of lines paper

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**Submission date:** 19-Aug-2020 07:15AM (UTC-0500)

**Submission ID:** 1371373996

**File name:** ides_using_coupled_mode_theory_and_the_method_of_lines_paper.pdf (548.9K)

**Word count:** 2196

**Character count:** 10514

# Analysis of Three Paralel Waveguides Using Coupled Mode Theory and the Method of Lines

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*Abstract*—**We present a numerical comparison of three parallel waveguides coupler characteristics by using the coupled mode theory and the method of lines. We analyzed for cases where a beam is launched into an input waveguide at the center and an outer input in the three waveguide arrangements. The output is then calculated using two methods. In the coupled mode theory we use an analytical solution and compare the results with semi numerical method of lines. The results show that both analytical and numerical scheme give similar result for simple three waveguide coupler structures.**

***Index Terms*** **--- Three paralel waveguide, coupled mode theory, method of lines.**

## I. Introduction

Optical waveguide couplers play an important role in the construction of flexible and highly reliable optical communication networks. Directional couplers are widely used as passive and active optical devices in fibre and integrated optics [1], and form the basis of components such as switches, modulators and wavelength filters. In its simple arrangements a directional couplers consists of two or more evanescently coupled waveguides place in close proximity, whose separation is sufficiently small that power may be transferred between the modes propagating in the two or more guides through an interaction involving their evanescent fields.

In a conventional coupler, light exchanges sinusoidally between the two or more guides guides as it propagates. The required coupling coefficient is determined by the propagation constant difference between the two lowest order modes. However, all directional couplers have an intrinsic wavelength dependence in their coupling ratio, which is very sensitive to parameters such as guide width, guide separation, refractive index difference and coupling length [2]. Changes in these parameters can cause a large change in the power splitting ratio. To analyze this three-core coupled waveguide system, usually a coupled-mode formulation is used. In this calculation all waveguide parameters are assumed to be identical

The method of lines (MoL) has been proved to be a very useful tool for the analysis of general waveguide systems [5]. It is a semi analytical method, which the wave equation is discretized as far as necessary in the transverse direction and solved analytically in the longitudinal direction, which results in less computational effort. An accurate result can be obtained since the MoL behaves in a stationary fashion and convergence is monotonic [6]. Discontinuous fields can be described accurately because the interface conditions are included in the calculation. Furthermore, the MoL is relatively easy to implement using computer numerical methods.

In this paper we present an analysis of power exchange along the propagation path of three coupled optical couplers, with parameters suitable for silica-on-silicon waveguides. The initial analysis is performed using weak coupled mode theory, in which the physical device shapes are abstracted into suitable variations of the input beam at different input arm. A comparison between the prediction of the coupled mode theory and a beam propagation scheme based on the method of lines is then used to determine the validity of the method of lines approach.

## II. Theoretical Background

### A. Coupled Mode Theory

To analyse the structure, we first construct a suitable theoretical model. In the geometry studied here, we shall consider a coupler consisting of three waveguides with constant width and gap as shown in Figure 1.

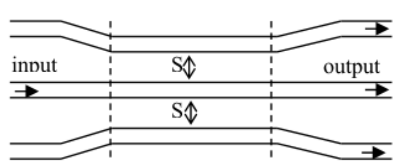



Figure 1. Three paralel waveguide sructure

The coupled-mode equations for a three-waveguide system are described as follows [3]:

$$\frac{d}{dz}\begin{bmatrix} a_1(z) \\ a_2(z) \\ a_3(z) \end{bmatrix} = -j \begin{bmatrix} \Delta\beta_1 & K & 0 \\ K & 0 & K \\ 0 & K & \Delta\beta_3 \end{bmatrix} \begin{bmatrix} a_1(z) \\ a_2(z) \\ a_3(z) \end{bmatrix} \qquad (1)$$

where :

$\Delta\beta_1 = \beta_1 - \beta_2$

$\Delta\beta_3 = \beta_3 - \beta_2$

$\beta_1$, $\beta_2$ and $\beta_3$ are the propagation constant of each waveguide, and $K_1$, $K_3$ is the coupling coefficients.

Since all waveguides are symmetry, $\Delta\beta_1 = \Delta\beta_3 = 0$, then the solution for Equation (1) is [3]:

$$\begin{bmatrix} a_1(z) \\ a_2(z) \\ a_3(z) \end{bmatrix} = \begin{bmatrix} c_1 + \frac{1}{2} & c_2 & c_1 - \frac{1}{2} \\ c_2 & 2c_1 & c_2 \\ c_1 - \frac{1}{2} & c_2 & c_1 + \frac{1}{2} \end{bmatrix} \begin{bmatrix} a_1(0) \\ a_2(0) \\ a_3(0) \end{bmatrix} \qquad (2)$$

where

$$c_1 = \frac{1}{2}\cos\sqrt{2}\,Kz$$

$$c_2 = -j\frac{1}{\sqrt{2}}\sin\sqrt{2}\,Kz$$

In three parallel waveguide we have two ways to launch the input input. First an input beam is launched into central waveguides, and secondly we can launch the input beam into one of the outer waveguide. For the first category the initial conditions become:

$$a_1(0) = a_3(0) = 0, \quad a_2(0) = 1 \qquad (3)$$

Then the solution for Equation (2) is :

$$a_1(z) = a_3(z) = c_2, \quad a_2(z) = 2c_1 \qquad (4)$$

The output power on each waveguide become :

$$P_1 = |a_1(z)|^2 = \frac{1}{2}\sin^2\sqrt{2}\,Kz \qquad (5)$$

$$P_2 = |a_2(z)|^2 = \cos^2\sqrt{2}\,Kz \qquad (6)$$

$$P_3 = |a_3(z)|^2 = \frac{1}{2}\sin^2\sqrt{2}\,Kz \qquad (7)$$

For the second category, a beam is launched into one of the outer waveguide (eg waveguide 1), the initial conditions are :

$$a_1(0) = 1, \quad a_2(0) = a_3(0) = 0 \qquad (8)$$

Then the solution for Equation (2) is :

$$a_1(z) = c_1 + \frac{1}{2}, \quad a_2(z) = c_2, \quad a_3(z) = c_1 - \frac{1}{2} \qquad (9)$$

In this case the output power become:

$$P_1 = |a_1(z)|^2 = \frac{1}{4}\cos^2\sqrt{2}\,Kz + \frac{1}{2}\cos\sqrt{2}\,Kz + \frac{1}{4} \qquad (10)$$

$$P_2 = |a_2(z)|^2 = \frac{1}{2}\sin^2\sqrt{2}\,Kz \qquad (11)$$

$$P_3 = |a_3(z)|^2 = \frac{1}{4}\cos^2\sqrt{2}\,Kz - \frac{1}{2}\cos\sqrt{2}\,Kz + \frac{1}{4} \qquad (12)$$

*B. The Method of Lines*

To solve three parallel waveguides coupler by the method of lines, the region under analysis is broken into a large number of small elements $\Delta x$ in the $x$ direction as shown in Figure 2.

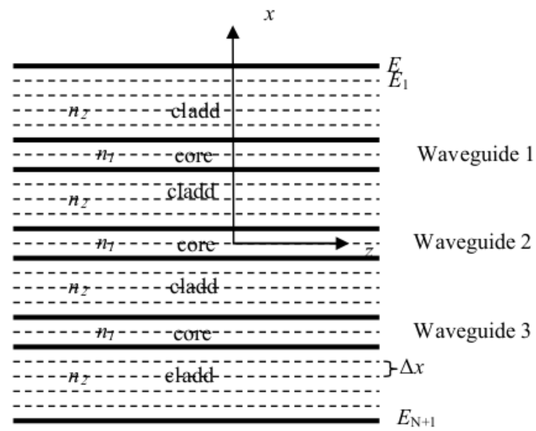



Figure 2. Discritization of three parallel waveguide structure

We begin by using Helmholtz equation for TE modes as [5-6],

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} + k^2 E_y = 0 \qquad (13)$$

A central difference approximation is then assumed for the second derivative term $d^2E/dx^2$, so that:

$$\frac{d^2 E_y}{dx^2} = \frac{E_{i+1} + 2E_1 + E_{i\ 1}}{\Delta x^2} \qquad (14)$$

If this is done, Equation (23) reduces to a matrix differential equation:

$$\frac{d^2\vec{E}_y}{dz^2} + \vec{Q}^2\ \vec{E} = 0 \tag{15}$$

where $\vec{E} = [E_1, E_2, E_3, ......, E_N]^t$ is a column vector containing discretised values of the field $E(x)$, at the points $x_1$, $x_2$, .... $x_N$, and $t$ on the bracket stands for transpose. The matrix $\vec{Q}$ can be written as [5-6]:

$$\vec{Q}^2 = \frac{1}{\Delta x^2}\begin{bmatrix} -2 & 1 & 0 & .. & 0 \\ 1 & -2 & .. & .. & .. \\ 0 & .. & .. & .. & 0 \\ .. & .. & .. & -2 & 1 \\ 0 & .. & 0 & 1 & -2 \end{bmatrix} + k_o^2 \begin{bmatrix} n_{x1} & 0 & .. & .. & 0 \\ 0 & n_{x2} & .. & .. & .. \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & 0 \\ 0 & .. & .. & 0 & n_{xN} \end{bmatrix} \tag{16}$$

In this equation there are always three components that are coupled with each other because of the tridiagonal structure of a matrix, so that a direct solution is not possible. A matrix transformation is therefore introduced such that,

$$\vec{\beta} = \vec{T}\ \vec{Q}\ \vec{T}^{\ 1} \tag{17}$$

Where $\vec{\beta}$ is diagonalization result from $\vec{Q}^2$ that consist Eigen value, and $\vec{T}$ consist Eigen vector from $\vec{Q}^2$.

Then, equation (25) can rewrite as :

$$\frac{d^2E_y}{dz^2} + \vec{\beta}^2\ \vec{E} = 0 \tag{18}$$

Equation (18) is wave equation that propagate in $z$ direction If we assume that no reflected waves occur (in, say, the particular example of a straight loss-less waveguide), therefore, Equation (18) has a solution as [5] :

$$\vec{E} = e^{-i\vec{\beta}z} \tag{19}$$

Therefore, total solution of wave that propagates along $z$ direction can be written as:

$$\vec{E} = \vec{T}\ e^{\ i\beta z}\ \vec{T}^{-1}\ \vec{E}_{inp} \tag{20}$$

One of the most important parameters associated with the waveguide is the fractional power that remains in the core at point $z$. This power is approximately given by the overlap integral:

$$P(z) = \left| \int_{\infty}^{\infty} E(x,0)\,E(x,z)\,dx \right|^2 \tag{21}$$

where $E(x,0)$ is the input field and $E(x,z)$ is the field at point $z$.

## III. Numerical Results

The light propagation at the coupling region significantly depends on the waveguide geometry such as propagation constant, refractive index and diameters of core and cladding.

Figure 3 shows wave propagation in three parallel waveguide as shown in Figure 1. In this simulation we use parameters such as core refractive index of 1.457, cladding refractive index of 1.463, wavelength of 1.52 µm, core width is 5 µm, and separation distance between waveguides is 5 µm.

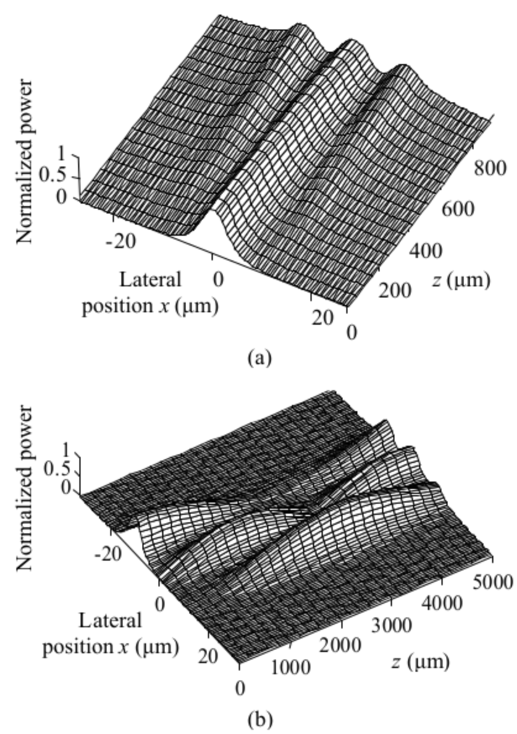



Figure 3. Wave propagation in three parallel waveguide (a) Input launched into central waveguide (b) Input launched into outer waveguide

If input launched into center waveguide, the light will coupled to two of outer waveguide, and the power is transferred. We see that the power of each waveguide become equal at $z = 940$ µm. In this case it can be used to design a directional coupler with equal output. However, if the input is launched into one of the outer waveguide, then the equal power output cannot be achieved.

Figure 4 show the mode profile of input and output. The input is launched into a center waveguide can be divided into three equal output power and happen at a distance $z = 940$ µm.

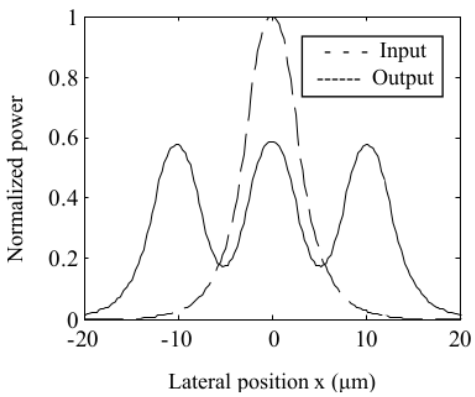
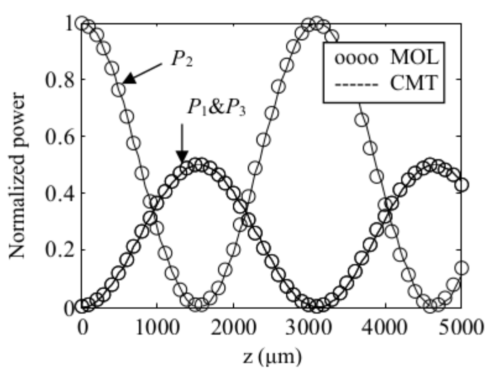


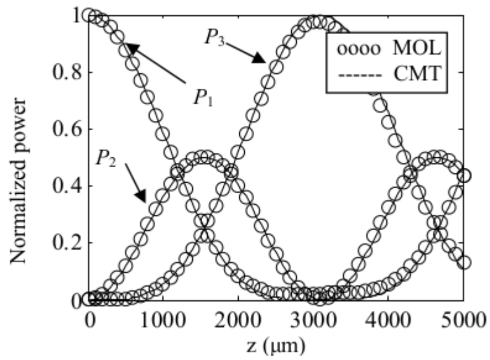

Figure 4. Power distribution in three parallel waveguide

## 5. Comparison between Coupled Mode Theory and Method of Lines

Figure 5 shows power along the propagation in three parallel waveguide. In this simulation we use two method. First coupled mode theory, the power in each waveguide can calculated by using Equation (15)-(17), (20)-(22). And second, the power were calculated by using the method of lines, where the output power were calculated by integrating field in each waveguide in along the distance $z$ as in Figure 3.




(a)




(b)

Figure 5. Power along the propagation in three parallel waveguide (a) Input launched into central waveguide (b) Input launched into outer waveguide

In the figure 5(a) all power of three waveguide meet periodically, it mean each waveguide have equal power. In figure 5(b) we don't find any meet point between three waveguide it means the power is never equal each other along the propagation.

As we see both methods produce similar results. The coupled mode theory as an analytical method is assumed to give the correct answer. Since the result of method of lines is similar to the coupled mode theory, this method has been proven to have good accuracy for simple three waveguides structures. However in term of calculation speed and simplicity, the coupled mode theory is simpler and easy to handle, meanwhile the calculation speed in the method of lines depends on discretization number of lines and hence bigger matrix size.

## 6. Conclusion

The analysis of three waveguide optical power splitter with identical and equally spaced waveguide has been done using two methods. We have demonstrated that in three parallel waveguide, if the wave was launched into the center waveguide it will coupled to two of outer waveguides and equal power output can be achieved. However if the light is launched into one of two outer waveguides, the power splitting will not be equal. These characteristics have been proven by the two methods. It shows that for simple three waveguides coupler both methods can be applied and produce similar results.


**Acknowledgement:**

Thank you for the financial support from Lembaga Penelitian dan Pengabdian Masyarakat University al Azhar Indonesia.

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